101 Shortcuts in Quantitative Aptitude

NUMBER SYSTEM

1. Method to multiply 2-digit number.
   (i) $AB \times CD = AC / AD + BC / BD$
   $35 \times 47 = 12 / 21 + 20 / 35 = 12 / 41 / 35 = 1645$
   (ii) $AB \times AC = A^2 / A(B + C) / BC$
   $74 \times 76 = 7^2 / 7(4 + 6) / 4 \times 6$
   $= 49 / 70 / 24 = 4^9 / 7^0 / 2^4 = 5624$
   (iii) $AB \times CC = AC / (A + B)C / BC$
   $= 35 \times 44 = 3 \times 4 / (3 + 5) \times 4 / 5 \times 4$
   $= 12 / 32 / 20 = 1^2 / 3^2 / 2^0 = 1540$
2. Method to multiply 3-digit no.
   $ABC \times DEF = AD / AE + BD / AF + BE + CD / BF + CE / CF$
   $456 \times 234 = 4 \times 2 / 4 \times 3 + 5 \times 2 / 4 \times 4 + 5 \times 3 + 6 \times 2 / 5 \times 4 + 6 \times 3 / 6 \times 4$
   $= 8 / 12 + 10 / 16 + 15 + 12 / 20 + 18 / 24$
   $= 8 / 2^2 / 4^3 / 3^8 / 2^4 = 106704$
3. If in a series all number contains repeating 7. To find their sum, we start from the left multiply 7 by 1, 2, 3, 4, 5 & 6. Look at the example below.
   $777777 + 77777 + 7777 + 777 + 77 + 7 = ?$
   $= 7 \times 1 / 7 \times 2 / 7 \times 3 / 7 \times 4 / 7 \times 5 / 7 \times 6$
   $= 7 / 1^4 / 2^1 / 2^8 / 3^5 / 4^2 = 864192$
4. $0.5555 + 0.555 + 0.55 + 0.5 = ?$
   To find the sum of those number in which one number is repeated after decimal, then first write the number in either increasing or decreasing order. Then -find the sum by using the below method.
   $0.5555 + 0.555 + 0.55 + 0.5$
   $= 5 \times 4 / 5 \times 3 / 5 \times 2 / 5 \times 1$
   $= 2^0 / 1^5 / 1^0 / 5 = 2.1605$
5. Those numbers whose all digits are 3.
   $(33)^2 = \underline{1}0\underline{8}9$ Those number. in which all digits are number is 3 two or more than 2 times repeated, to find the square of these number, we repeat 1 and 8 by (n – 1) time. Where n → Number of times 3 repeated.
   $(333)^2 = \underline{11}0\underline{88}9$
   $(3333)^2 = \underline{111}0\underline{888}9$
6. Those number whose all digits are 9.
   $(99)^2 = 9801$
   $(999)^2 = 998001$
   $(9999)^2 = 99980001$
   $(99999)^2 = 9999800001$
7. Those number whose all digits are 1.
   A number whose one's, ten's, hundred's digit is 1 i.e., 11, 111, 1111, ...
   In this we count number of digits. We write 1, 2, 3, ... in their square the digit in the number, then write in decreasing order up to 1.
   $11^2 = 121$
   $111^2 = 12321$
   $1111^2 = 1234321$

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8. Some properties of square and square root:
   (i) Complete square of a no. is possible if its last digit is 0, 1, 4, 5, 6 & 9. If last digit of a no. is 2, 3, 7, 8 then complete square root of this no. is not possible.
   (ii) If last digit of a no. is 1, then last digit of its complete square root is either 1 or 9.
   (iii) If last digit of a no. is 4, then last digit of its complete square root is either 2 or 8.
   (iv) If last digit of a no. is 5 or 0, then last digit of its complete square root is either 5 or 0.
   (v) If last digit of a no. is 6, then last digit of its complete square root is either 4 or 6.
   (vi) If last digit of a no. is 9, then last digit of its complete square root is either 3 or 7.

9. **Prime Number :**
   (i) Find the approx square root of given no. Divide the given no. by the prime no. less than approx square root of no. If given no. is not divisible by any of these prime no. then the no. is prime otherwise not.

   For example : To check 359 is a prime number or not.

   Sol. Approx sq. root = 19
   Prime no. < 19 are 2, 3, 5, 7, 11, 13, 17
   359 is not divisible by any of these prime nos. So 359 is a prime no.
   For example: Is $2^{5001} + 1$ is prime or not?

   $$\frac{2^{5001}+1}{2+1} \Rightarrow \text{Reminder} = 0,$$

   $\therefore$ $2^{5001} + 1$ is not prime.
   (ii) There are 15 prime no. from 1 to 50.
   (iii) There are 25 prime no. from 1 to 100.
   (iv) There are 168 prime no. from 1 to 1000.

10. If a no. is in the form of $x^n + a^n$, then it is divisible by $(x + a)$; if n is odd.
11. If $x^n \div (x - 1)$, then remainder is always 1.
12. If $x^n \div (x + 1)$
   (i) If n is even, then remainder is 1.
   (ii) If n is odd, then remainder is x.

13. (i) Value of $\sqrt{P+\sqrt{P+\sqrt{P+.....\infty}}} = \frac{\sqrt{4P+1}+1}{2}$

   (ii) Value of $\sqrt{P-\sqrt{P-\sqrt{P-.....\infty}}} = \frac{\sqrt{4P+1}-1}{2}$

   (iii) Value of $\sqrt{P.\sqrt{P.\sqrt{P.....\infty}}} = P$

   (iv) Value of $\sqrt{P\sqrt{P\sqrt{P\sqrt{P\sqrt{P}}}}} = P^{\left(2^n-1\right)\div 2^n}$

   [Where $n \rightarrow$ no. of times P repeated].

   Note: If factors of P are n & (n + 1) type then value of $\sqrt{P+\sqrt{P+\sqrt{P+....\infty}}} = (n+1)$ and $\sqrt{P-\sqrt{P-\sqrt{P-....\infty}}} = n$.

14. **Number of divisors :**
   (i) If N is any no. and $N = a^n \times b^m \times c^p \times ....$ where a, b, c are prime no.
   No. of divisors of $N = (n + 1)(m + 1)(p + 1)....$
   e.g. Find the no. of divisors of 90000.
   $N = 90000 = 2^2 \times 3^2 \times 5^2 \times 10^2 = 2^2 \times 3^2 \times 5^2 \times (2 \times 5)^2 = 2^4 \times 3^2 \times 5^4$
   So, the no. of divisors $= (4 + 1)(2 + 1)(4 + 1) = 75$
   (ii) $N = a^n \times b^m \times c^p$, where a, b, c are prime
   Then set of co-prime factors of $N = [(n + 1)(m + 1)(p + 1) - 1 + nm + mp + pn + 3mnp]$

   (iii) If $N = a^n \times b^m \times c^p ...$, where a, b & c are prime no. Then sum of the divisors $= \frac{\left(a^{n+1}-1\right)\left(b^{m+1}-1\right)\left(c^{p+1}-1\right)}{(a-1)(b-1)(c-1)}$

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15. **To find the last digit or digit at the unit's place of a^n.**

(i) If the last digit or digit at the unit's place of a is 1, 5 or 6, whatever be the value of n, it will have the same digit at unit's place, i.e.,

$(.....1)^n = (.....1)$
$(.....5)^n = (.....5)$
$(.....6)^n = (.....6)$

(ii) If the last digit or digit at the units place of a is 2, 3, 5, 7 or 8, then the last digit of a^n depends upon the value of n and follows a repeating pattern in terms of 4 as given below :

n	last digit of $(....2)^n$	last digit of $(....3)^n$	last digit of $(....7)^n$	last digit of $(....8)^n$
4x+1	2	3	7	8
4x+2	4	9	9	4
4x+3	8	7	3	2
4x	6	1	1	6

(iii) If the last digit or digit at the unit's place of a is either 4 or 9, then the last digit of a^n depends upon the value of n and follows repeating pattern in terms of 2 as given below.

n	last digit of $(....4)^n$	last digit of $(....9)^n$
2x	6	1
2x + 1	4	9

16. (i) Sum of n natural number $= \frac{(n)(n+1)}{2}$

(ii) Sum of n even number = (n) (n + 1)

(iii) Sum of n odd number = n^2

17. (i) Sum of sq. of first n natural no. = $\frac{n(n+1)(2n+1)}{6}$

(ii) Sum of sq. of first n odd natural no. = $\frac{n\left(4n^2-1\right)}{3}$

(iii) Sum of sq. of first n even natural no. = $\frac{2n(n+1)(2n+1)}{3}$

18. (i) Sum of cube of first n natural no. = $\frac{n^2(n+1)^2}{4} = \left[\frac{n(n+1)}{2}\right]^2$

(ii) Sum of cube of first n even natural no. = $2n^2 (n + 1)^2$

(iii) Sum of cube of first n odd natural no. = $n^2 (2n^2 – 1)$

19. (i) $x^n – y^n$ is divisible by (x + y)
When n is even

(ii) $x^n – y^n$ is divisible by (x – y)
When n is either odd or even.

20. For any integer n, $n^3 – n$ is divisible by 3, $n^5 – n$ is divisible by 5, $n^{11} – n$ is divisible by 11, $n^{13} – n$ is divisible by 13.

21. **Some articles related to Divisibility :**

(i) A no. of 3-digits which is formed by repeating a digit 3-times, then this no. is divisible by 3 and 37.
e.g., 111, 222, 333, ...

(ii) A no. of 6-digit which is formed by repeating a digit 6-times then this no. is divisible by 3, 7, 11, 13 and 37.
e.g., 111111, 222222, 333333, 444444, ...

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22. **Divisible by 7 :** We use osculator (– 2) for divisibility test.
99995 : 9999 – 2 × 5 = 9989
9989 : 998 – 2 × 9 = 980
980 : 98 – 2 × 0 = 98
Now 98 is divisible by 7, so 99995 is also divisible by 7.

23. **Divisible by 11 :** In a number, if difference of sum of digit at even places and sum of digit at odd places is either 0 or multiple of 11, then no. is divisible by 11.
For example, 12342 ÷ 11
Sum of even place digit = 2 + 4 = 6
Sum of odd place digit = 1 + 3 + 2 = 6
Difference = 6 – 6 = 0
∴ 12342 is divisible by 11.

24. **Divisible by 13 :** We use (+ 4) as osculator.
e.g., 876538 ÷ 13
876538: 8 × 4 + 3 = 35
5 × 4 + 3 + 5 = 28
8 × 4 + 2 + 6 = 40
0 × 4 + 4 + 7 = 11
1 × 4 + 1 + 8 = 13
13 is divisible by 13.
∴ 876538 is also divisible by 13.

25. **Divisible by 17 :** We use (– 5) as osculator.
e.g., 294678: 29467 – 5 × 8 = 29427
27427: 2942 – 5 × 7 = 2907
2907: 290 – 5 × 7 = 255
255: 25 – 5 × 5 = 0
∴ 294678 is completely divisible by 17.

26. **Divisible by 19 :** We use (+ 2) as osculator.
e.g: 149264: 4 × 2 + 6 = 14
4 × 2 + 1 + 2 = 11
1 × 2 + 1 + 9 = 12
2 × 2 + 1 + 4 = 9
9 × 2 + 1 = 19
19 is divisible by 19
∴ 149264 is divisible by 19.

27. **HCF (Highest Common factor)**
There are two methods to find the HCF–
(a) Factor method (b) Division method
(i) For two no. a and b if a < b, then HCF of a and b is always less than or equal to a .
(ii) The greatest number by which x, y and z completely divisible is the HCF of x, y and z.
(iii) The greatest number by which x, y, z divisible and gives the remainder a, b and c is the HCF of (x –a), (y–b) and (z–c).
(iv) The greatest number by which x, y and z divisible and gives same remainder in each case, that number is HCF of (x–y), (y–z) and (z–x).
(v) H.C.F. of $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f} = \frac{\text{H.C.M. of (a, c, e)}}{\text{L.C.M. of (b, d, f)}}$

28. **LCM (Least Common Multiple)**
There are two methods to find the LCM–
(a) Factor method (b) Division method
(i) For two numbers a and b if a < b, then L.C.M. of a and b is more than or equal to b.
(ii) If ratio between two numbers is a : b and their H.C.F. is x, then their L.C.M. = abx.

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(iii) If ratio between two numbers is a : b and their L.C.M. is x, then their H.C.F. $= \frac{x}{ab}$

(iv) The smallest number which is divisible by x, y and z is L.C.M. of x, y and z.

(v) The smallest number which is divided by x, y and z give remainder a, b and c, but (x – a) = (y – b) = (z – c) = k, then number is (L.C.M. of (x, y and z) – k).

(vi) The smallest number which is divided by x, y and z give remainder k in each case, then number is (L.C.M. of x, y and z) + k.

(vii) L.C.M. of $\frac{a}{b}, \frac{c}{d}$ and $\frac{e}{f} = \frac{\text{L.C.M. of (a, c, e)}}{\text{H.C.F. of (b, d, f)}}$

(viii) For two numbers a and b –
LCM × HCF = a × b

(ix) If a is the H.C.F. of each pair from n numbers and L is L.C.M., then product of n numbers = $a^{n-1}.L$

ALGEBRA

29. **Algebra Identities:**

(i) $(a + b)^2 + (a – b)^2 = 2 (a^2 + b^2)$

(ii) $(a + b)^2 – (a – b)^2 = 4ab$

(iii) $a^3 + b^3 = (a + b) (a^2 – ab + b^2)$

(iv) $a^3 – b^3 = (a – b) (a^2 + ab + b^2)$

(v) $a^4 + a^2 + 1 = (a^2 + a + 1) (a^2 – a + 1)$

(vi) If a + b + c = 0, then $a^3 + b^3 + c^3 = 3abc$

(vii) $\frac{(a+b)^2 - (a-b)^2}{ab} = 4$

(viii) $\frac{(a+b)^2 + (a-b)^2}{a^2 + b^2} = 2$

(ix) $a\frac{b}{c} + d\frac{e}{f} + g\frac{h}{i} - j\frac{k}{l} = (a + d + g - j) + \left(\frac{b}{c} + \frac{e}{f} + \frac{h}{i} - \frac{k}{l}\right)$

(x) If a + b + c = abc, then

$$\left(\frac{2a}{1-a^2}\right) + \left(\frac{2b}{1-b^2}\right) + \left(\frac{2c}{1-c^2}\right) = \left(\frac{2a}{1-a^2}\right) \cdot \left(\frac{2b}{1-b^2}\right) \cdot \left(\frac{2c}{1-c^2}\right) \text{ and}$$

$$\left(\frac{3a-a^3}{1-3a^2}\right) + \left(\frac{3b-b^3}{1-3b^2}\right) + \left(\frac{3c-c^3}{1-3c^2}\right) = \left(\frac{3a-a^3}{1-3a^2}\right) \cdot \left(-\frac{3b-b^3}{1-3b^2}\right) \cdot \left(\frac{3c-c^3}{1-3c^2}\right)$$

30. If $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$, then

(i) If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, one solution.

(ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, Infinite many solutions.

(iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, No solution

31. If α and β are roots of $ax^2 + bx + c = 0$, then $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ are roots of $cx^2 + bx + a = 0$

32. If α and β are roots of $ax^2 + bx + c = 0$, then

(i) One root is zero if c = 0.

(ii) Both roots zero if b = 0 and c = 0.

(iii) Roots are reciprocal to each other, if c = a.

(iv) If both roots α and β are positive, then sign of a and b are opposite and sign of c and a are same.

(v) If both roots α and β are negative, then sign of a, b and c are same.

$(\alpha + \beta) = -\frac{b}{a}, \alpha\beta = \frac{c}{a}$, then

$$\alpha - \beta = \sqrt{(\alpha + \beta)^2 - 4\alpha\beta}$$

$$\alpha^4 + \beta^4 = \left(\alpha^2 + \beta^2\right)^2 - 2\alpha^2\beta^2 = \left[(\alpha + \beta)^2 - 2\alpha\beta\right]^2 - 2(\alpha\beta)^2$$

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33. **Arithmetic Progression:**

(i) If a, a + d, a + 2d, ... are in A.P., then, nth term of A.P. $a_n = a + (n-1)d$

Sum of n terms of this A.P. = $S_n = \frac{n}{2}\left[2a + (n-1)d\right] = \frac{n}{2}[a + l]$ where ℓ = last term

a = first term
d = common difference

(ii) A.M. = $\frac{a+b}{2}$ [$\because$ A.M. = Arithmetic mean]

34. **Geometric Progression:**

(i) G.P. $\rightarrow$ a, ar, ar^2,.....
Then, nth term of G.P. $a_n = ar^{n-1}$

$$S_n = \frac{a\left(r^n - 1\right)}{(r-1)}, r > 1$$

$$= \frac{a(1-r^n)}{(1-r)}, r < 1$$

$S_\infty = 1 - r$ [where r = common ratio, a = first term]

(ii) G.M. = $\sqrt{ab}$

35. If a, b, c are in H.P., $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.

n^{th} term of H.M. = $\frac{1}{n^{th} \text{ term of A.P.}}$

$$\text{H.M.} = \frac{2ab}{a+b}$$

Note : Relation between A.M., G.M. and H.M.

(i) A.M. × H.M. = $G.M.^2$

(ii) A.M. > G.M. > H.M.
A.M. $\rightarrow$ Arithmetic Mean
G.M. $\rightarrow$ Geometric Mean
H.M. $\rightarrow$ Harmonic Mean

AVERAGE

36. (i) Average of first n natural no. = $\frac{n+1}{2}$

(ii) Average of first n even no. = (n + 1)

(iii) Average of first n odd no. = n

37. (i) Average of sum of square of first n natural no. = $\frac{(n+1)(2n+1)}{6}$

(ii) Average of sum of square of first n even no. = $\frac{2(n+1)(2n+1)}{3}$

(iii) Average of sum of square of first odd no. = $\left(\frac{4n^2-1}{3}\right)$

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38. (i) Average of cube of first n natural no. = $\frac{n(n+1)^2}{4}$

(ii) Average of cube of first n even natural no. = $2n(n+1)^2$

(iii) Average of cube of first n odd natural no. = $n(2n^2-1)$

39. Average of first n multiple of m = $\frac{m(n+1)}{2}$

40. (i) If average of some observations is x and a is added in each observations, then new average is (x + a).

(ii) If average of some observations is x and a is subtracted in each observations, then new average is (x – a).

(iii) If average of some observations is x and each observations multiply by a, then new average is ax.

(iv) If average of some observations is x and each observations is divided by a, then new average is $\frac{x}{a}$.

(v) If average of n_1 is A_1, & average of n_2 is A_2, then Average of $(n_1 + n_2)$ is $\frac{n_1A_1 + n_2A_2}{n_1 + n_2}$ and

Average of $(n_1 - n_2)$ is $\frac{n_1A_1 - n_2A_2}{n_1 - n_2}$

41. When a person is included or excluded the group, then age/weight of that person = No. of persons in group × (Increase / Decrease) in average $\pm$ New average.

For example : In a class average age of 15 students is 18 yrs. When the age of teacher is included their average increased by 2 yrs, then find the age of teacher.

Sol. Age of teacher = 15 × 2 + (18 + 2) = 30 + 20 = 50 yrs.

42. When two or more than two persons included or excluded the group, then average age of included or excluded person is

$$= \frac{\text{No. of person} \times (\text{Increase / Decrease}) \text{in average} \pm \text{New average} \times (\text{No. of person included or excluded})}{\text{No. of included or person}}$$

For example : Average weight of 13 students is 44 kg. After including two new students their average weight becomes 48 kg, then find the average weight of two new students.

Sol. Average weight of two new students

$$= \frac{13 \times (48-44) + 48 \times 2}{2} = \frac{13 \times 4 + 48 \times 2}{2} = \frac{52+96}{2} = 74 \text{ kg}$$

43. If a person travels two equal distances at a speed of x km/h and y km/h, then average speed = $\frac{2xy}{x+y}$ km/h

44. If a person travels three equal distances at a speed of x km/h, y km/h and z km/h, then average speed = $\frac{3xyz}{xy+yz+zx}$ km/h.

RATIO & PROPORTION

45. (i) If $\frac{a}{K_1} = \frac{b}{K_2} = \frac{c}{K_3} = ... ,$ then $\frac{a+b+c+....}{c} = \frac{K_1+K_2+K_3+.....}{K_3}$

For example: If $\frac{P}{3} = \frac{Q}{4} = \frac{R}{7}$, then find $\frac{P+Q+R}{R}$

Sol. P = 3, Q = 4, R = 7

Then $\frac{P+Q+R}{R} = \frac{3+4+7}{7} = 2$

(ii) If $\frac{a_1}{a_2} = \frac{a_2}{a_3} = \frac{a_3}{a_4} = \frac{a_4}{a_5} = ... \frac{a_n}{a_{n+1}} = K$, then $a_1 : a_{n+1} = (K)^n$

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46. A number added or subtracted from a, b, c & d, so that they are in proportion = $\frac{ad - bc}{(a+d)-(b+c)}$

For example : When a number should be subtracted from 2, 3, 1 & 5 so that they are in proportion. Find that number.

Sol. Req. No. = $\frac{2\times5-3\times1}{(2+5)-(3+1)} = \frac{10-3}{7-4} = \frac{7}{3}$

47. If X part of A is equal to Y part of B, then A : B = Y : X.
For example: If 20% of A = 30% of B, then find A : B.

Sol. A : B = $\frac{30\%}{20\%} = \frac{3}{2}$ = 3 : 2

48. When X^{th} part of P, Y^{th} part of Q and Z^{th} part of R are equal, then find A : B : C.
Then, A : B : C = yz : zx : xy

TIME, DISTANCE AND WORK

49. A can do a/b part of work in t_1 days and c/d part of work in t_2 days, then $\frac{t_1}{a/b} = \frac{t_2}{c/d}$

50. (i) If A is K times efficient than B, Then $T(K + 1) = Kt_B$
(ii) If A is K times efficient than B and takes t days less than B

Then T = $\frac{Kt}{K^2-1}$ or $\frac{t}{K-1}$, $t_B = \frac{t}{K-1} = kt_A$

51. (i) If a cistern takes X min to be filled by a pipe but due to a leak, it takes Y extra minutes to be filled, then the time taken by leak to empty the cistern = $\left(\frac{X^2+XY}{Y}\right)$min

(ii) If a leak empty a cistern in X hours. A pipe which admits Y litres per hour water into the cistern and now cistern is emptied in Z hours, then capacity of cistern is = $\left(\frac{X+Y+Z}{Z-X}\right)$ litres.

(iii) If two pipes A and B fill a cistern in x hours and y hours. A pipe is also an outlet C. If all the three pipes are opened together, the tank full in T hours. Then the time taken by C to empty the full tank is = $\left[\frac{xyT}{yT+xT-xy}\right]$

52. (i) If t_1 and t_2 time taken to travel from A to B and B to A, with speed *a* km/h and *b* km/h, then distance from A to B is

$d = (t_1 + t_2)\left(\frac{ab}{a+b}\right)$ $d = (t_1 - t_2)\left(\frac{ab}{a-b}\right)$

$d = (a-b)\left(\frac{t_1t_2}{t_1-t_2}\right)$

(ii) If Ist part of distance is covered at the speed of *a* in t_1 time and the second part is covered at the speed of *b* in t_2 time, then the average speed = $\left(\frac{at_2+bt_1}{t_1+t_2}\right)$

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PERCENTAGE

53.

Simple Fraction	Their Percentage
1	100%
$\frac{1}{2}$	50%
$\frac{1}{3}$	33.3%
$\frac{1}{4}$	25%
$\frac{1}{5}$	20%
$\frac{1}{6}$	16.67%
$\frac{1}{7}$	14.28%

Simple Fraction	Their Percentage
$\frac{1}{8}$	12.5%
$\frac{1}{9}$	11.11%
$\frac{1}{10}$	10%
$\frac{1}{11}$	9.09%
$\frac{1}{12}$	8.33%

54. (i) If A is $\left(x\% = \frac{a}{b}\right)$ more than B, then B is $\left(\frac{a}{a+b}\%\right)$ less than A.

(ii) If A is $\left(x\% = \frac{a}{b}\right)$ less than B, then B is $\left(\frac{a}{a-b}\%\right)$ more than A

if a > b, we take a – b

if b > a, we take b – a.

55. If price of a article increase from ₹ a to ₹ b, then its expenses decrease by $\left(\frac{b-a}{b} \times 100\right)\%$ so that expenditure will be same.

56. Due to increase/decrease the price x%, A man purchase *a* kg more in ₹ y, then

Per kg increase or decrease = $\left(\frac{xy}{100 \times a}\right)$

Per kg starting price = ₹ $\frac{xy}{(100 \pm x)a}$

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57. For two articles, if price:

Ist	IInd	Overall
Increase (x%)	Increase (y%)	Increase $\left(x+y+\frac{xy}{100}\right)\%$
Increase (x%)	Decrease (y%)	$\left(x-y-\frac{xy}{100}\right)\%$ If +ve (Increase) If –ve (Decrease)
Decrease (x%)	Decrease (y%)	Decrease $\left(x+y-\frac{xy}{100}\right)\%$
Increase (x%)	Decrease (x%)	Decrease $\left(\frac{x^2}{100}\right)\%$

58. If the side of a square or radius of a circle is x% increase/decrease, then its area increase/decrease = $\left(2x \pm \frac{x^2}{100}\right)\%$

59. If the side of a square, x% increase/decrease then x% its perimeter and diagonal increase/decrease.

60. (i) If population P increase/decrease at r% rate, then after t years population = $P\left(\frac{100 \pm R}{100}\right)^t$

(ii) If population P increase/decrease r_1% first year, r_2% increase/decrease second year and r_3% increase/decrease third year, then after 3 years population = $P\left(1 \pm \frac{r_1}{100}\right)\left(1 \pm \frac{r_2}{100}\right)\left(1 \pm \frac{r_3}{100}\right)$

If increase we use (+), if decrease we use (–)

61. If a man spend x% of this income on food, y% of remaining on rent and z% of remaining on cloths. If he has ₹ P remaining, then total income of man is = $\frac{P \times 100 \times 100 \times 100}{(100-x)(100-y)(100-z)}$

[**Note:** We can use this table for area increase/decrease in mensuration for rectangle, triangle and parallelogram].

PROFIT AND LOSS

62. If CP of *x* things = SP of *y* things, then

Profit/Loss = $\left[\frac{x-y}{y} \times 100\right]\%$

If +ve, Profit;
If –ve, Loss

63. If after selling *x* things P/L is equal to SP of *y* things,

then P/L = $\frac{y}{(x \pm y)} \times 100$

$\begin{bmatrix} \text{Profit} = - \\ \text{Loss} = + \end{bmatrix}$

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64. If CP of two articles are same, and they sold at

Ist	IInd	Overall
(x%) Profit	(y%) Profit	$\left(\frac{x+y}{2}\right)\%$ Profit
(x%) Profit	(y%) Loss	$\left(\frac{x-y}{2}\right)\%\begin{cases}\text{Profit, if } x > y\\ \text{Loss, if } x < y\end{cases}$
(x%) Loss	(y%) Loss	$\left(\frac{x+y}{2}\right)\%$ Loss
(x%) Profit	(y%) Loss	No profit, no loss

65. If SP of two articles are same and they sold at

Ist	IInd	Overall
Profit (x%)	Loss(x%)	$\text{Loss}\left(\frac{x^2}{100}\right)\%$
Profit (x%)	Loss (y%)	$\left(\frac{100(x-y)-2xy}{200+x-y}\right)\%$ or $\left[\frac{2(100+x)(100-y)}{200+x-y}-100\right]\%\begin{cases}\text{If + ve, then Profit\%}\\ \text{If − ve, then Loss\%}\end{cases}$

66. After D% discount, requires P% profit, then total increase in C.P.= $\left[\frac{P+D}{100-D}\times 100\right]\%$

67. M.P. = C.P × $\frac{(100+P)}{(100-D)}$

68. Profit % = $\frac{(\text{M.P.} - \text{C.P.})\times 100}{\text{C.P.}}$

69. (i) For discount r_1% and r_2%, successive discount $=\left[\left(\frac{100+r_1}{100}\right)\left(\frac{100+r_2}{100}\right)\left(\frac{100+r_3}{100}\right)-1\right]\times 100$

(ii) For discount r_1%, r_2% and r_3%, successive discount $=\left[\left(\frac{100+r_1}{100}\right)\left(\frac{100+r_2}{100}\right)\left(\frac{100+r_3}{100}\right)-1\right]\times 100$

SIMPLE AND COMPOUND INTEREST

If P = Principal, R = Rate per annum,
T = Time in years, SI = Simple interest,
A = Amount

70. (i) $SI = \frac{PRT}{100}$

(ii) $A = P + SI = P\left[1+\frac{RT}{100}\right]$

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71. If P = Principal, A = Amount
in n years, R = rate of interest per annum.

$A = P\left[1+\frac{R}{100}\right]^n$, interest payable annually

72. (i) $A = P\left[1+\frac{R'}{100}\right]^{n'}$, interest payable half-yearly

$R' = R/2, n' = 2n$

(ii) $A = P\left[1+\frac{R}{400}\right]^{4n}$, interest payable quarterly;

73. (i) $\left[1+\frac{R}{400}\right]$ is the yearly growth factor;

(ii) $\left[1-\frac{R}{400}\right]$ is the yearly decay factor or depreciation factor.

74. When time is fraction of a year, say $4\frac{3}{4}$, years, then,

$$\text{Amount} = P\left[1+\frac{R}{100}\right]^4 \times \left[1+\frac{\frac{3}{4}R}{100}\right]$$

75. $CI = \text{Amount} - \text{Principal} = P\left[\left(1+\frac{R}{100}\right)^n - 1\right]$

76. When Rates are different for different years, say R_1, R_2, R_3% for 1st, 2nd & 3rd years respectively, then,

77. $\text{Amount} = P\left[1+\frac{R_1}{100}\right]\left[1+\frac{R_2}{100}\right]\left[1+\frac{R_3}{100}\right]$

In general, interest is considered to be SIMPLE unless otherwise stated.

GEOMETRY

78. (i) Sum of all the exterior angle of a polygon = 360°

(ii) Each exterior angle of a regular polygon = $\frac{360°}{n}$

(iii) Sum of all the interior angles of a polygon = $(n-2) \times 180°$

(iv) Each interior angle of a regular polygon = $\frac{(n-2)}{n} \times 180°$

(v) No. of diagonals of a polygon = $\frac{n(n-3)}{2}$, $n \rightarrow$ no. of sides.

(vi) The ratio of sides a polygon to the diagonals of a polygon is $2 : (n-3)$

(vii) Ratio of interior angle to exterior angle of a regular polygon is $(n-2) : 2$

79. **Properties of triangle:**

(i) When one side is extended in any direction, an angle is formed with another side. This is called the exterior angle. There are six exterior angles of a triangle.

(ii) Interior angle + corresponding exterior angle = 180°.

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(iii) An exterior angle = Sum of the other two interior opposite angles.

(iv) Sum of the lengths of any two sides is greater than the length of third side.

(v) Difference of any two sides is less than the third side.
Side opposite to the greatest angle is greatest and vice versa.

(vi) A triangle must have at least two acute angles.

(vii) Triangles on equal bases and between the same parallels have equal areas.

(viii) If a, b, c denote the sides of a triangle then

(i) if $c^2 < a^2 + b^2$, Triangle is acute angled.

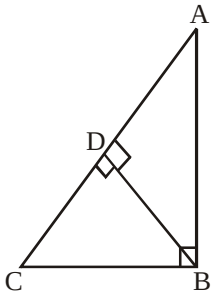
(ii) if $c^2 = a^2 + b^2$, Triangle is right angled.

(iii) if $c^2 > a^2 + b^2$, Triangle is obtuse angled.

(ix) If 2 triangles are equiangular, their corresponding sides are proportional. In triangles ABC and XYZ, if $\angle A = \angle X, \angle B = \angle Y, \angle C = \angle Z$, then

$$\frac{AB}{XY} = \frac{AC}{XZ} = \frac{BC}{YZ} \cdot \cdot$$

- (i) In $\Delta ABC, \angle B = 90°$ $BD \perp AC$
$\therefore BD \times AC = AB \times BC$



(ii) $$\frac{1}{BD^2} = \frac{1}{AB^2} + \frac{1}{BC^2}$$

(iii) $BD^2 = AD \times DC$

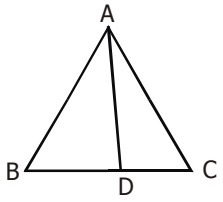
(x) The perpendiculars drawn from vertices to opposite sides (called altitudes) meet at a point called **Orthocentre** of the triangle.

(xi) The line drawn from a vertex of a triangle to the opposite side such that it bisects the side is called the **Median** of the triangle. A median bisects the area of the triangle.

(xii) When a vertex of a triangle is joined to the midpoint of the opposite side, we get a median. The point of intersection of the medians is called the **Centroid** of the triangle. The centroid divides any median in the ratio 2 : 1.

(xiii) Angle Bisector Theorem–

In the figure if AD is the angle bisector (interior) of $\angle BAC$. Then,



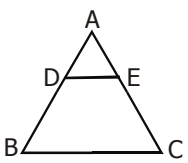
1. $AB/AC = BD/DC$.
2. $AB \times AC - BD \times DC = AD^2$.

(xiv) Midpoint Theorem –

In a triangle, the line joining the mid points of two sides is parallel to the third side and half of it.

(xv) Basic Proportionality Theorem

A line parallel to any one side of a triangle divides the other two sides proportionally. If DE is parallel to BC, then



$$\frac{AD}{BD} = \frac{AE}{EC'} \frac{AB}{AD} = \frac{AC}{AE}, \frac{AD}{DE} = \frac{AB}{BC}$$ and so on.

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80. **Properties of circle –**

(i) Only one circle can pass through three given points.

(ii) There is one and only one tangent to the circle passing through any point on the circle.

(iii) From any exterior point of the circle, two tangents can be drawn on to the circle.

(iv) The lengths of two tangents segment from the exterior point to the circle, are equal.

(v) The tangent at any point of a circle and the radius through the point are perpendicular to each other.

(vi) When two circles touch each other, their centres & the point of contact are collinear.

(vii) If two circles touch externally, distance between centres = sum of radii.

(viii) If two circles touch internally, distance between centres = difference of radii

(ix) Circles with same centre and different radii are **concentric circles**.

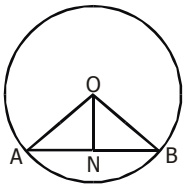
(x) Points lying on the same circle are called **concyclic points**.

(xi) Measure of an arc means measure of central angle.
m(minor arc) + m(major arc) = 360°.

(xii) Angle in a semicircle is a right angle.

(xiii) Only one circle can pass through three given

(xxv) If ON is ⊥ from the centre O of a circle to a chord AB, then AN = NB.



(⊥ from centre bisects chord)

(xv) If N is the midpoint of a chord AB of a circle with centre O, then ∠ONA = 90°.
(Converse, ⊥ from centre bisects chord)

(xvi) Two congruent figures have equal areas but the converse need not be true.

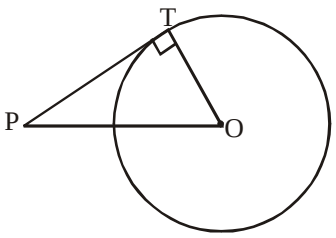
(xvii) A diagonal of a parallelogram divides it into two triangles of equal area.

(xviii) Parallelograms on the same base and between the same parallels are equal in area.

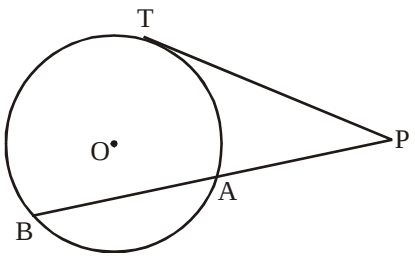
(xix) Triangles on the same bases and between the same parallels are equal in area.

(xx) If a triangle and a parallelogram are on the same base and between the same parallels, then the area of the triangle is equal to the half of the parallelogram.

- If PT is a tangent to the circle, then $OP^2 = PT^2 = OT^2$



- If PT is tangent and PAB is secant of a circle, then $PT^2 = PA.PB$



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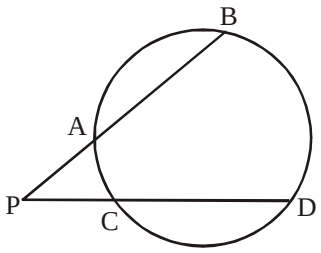
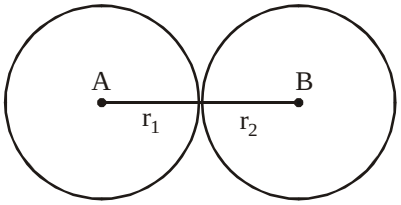
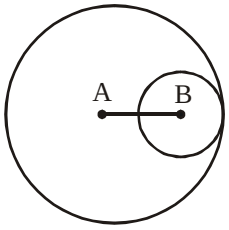
- If PB & PD are two secant of a circle, then PA.PB = PC.PD



- If two circles touch externally, then distance between their centres = $(r_1 + r_2)$



- If two circles touch internally, then distance between their centres = $r_1 - r_2$ where $r_1 > r_2$.



MENSURATION

81. (i) Area of triangle = $\frac{1}{2}$ × base × altitude

(ii) Area of triangle using heron's formula = $\sqrt{S/S-a\,(S-b)\,(S-c)}$, where S = $\frac{a+b+c}{2}$

82. In an equilateral triangle with side *a*, then

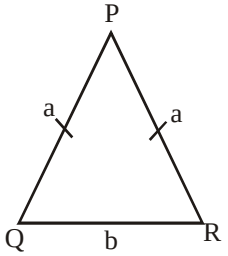
$$\boxed{\frac{4A}{\sqrt{3}} = \frac{4h^2}{3} = \frac{P^2}{9} = a^2}$$

where A → Area of triangle
P → Perimeter
h → Height

83. In an isosceles triangle PQR

ar Δ PQR = $\frac{b}{4}\sqrt{4a^2 - b^2}$

Height = $\sqrt{\frac{4a^2 - b^2}{2}}$



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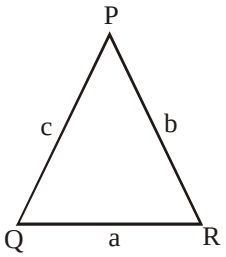
84. (i) Area of $\Delta = \frac{1}{2}bc\ SinP$ where $\angle P = \angle QPR$

(ii) Area of $\Delta = \frac{1}{2}ac\ SinQ$

(iii) Area of $\Delta = \frac{1}{2}ab\ SinR$

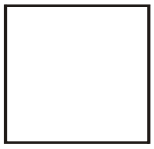


85. $CosP = \frac{b^2 + c^2 - a^2}{2bc}$, $CosQ = \frac{a^2 + c^2 - b^2}{2ac}$,

$CosR = \frac{a^2 + b^2 - c^2}{2ab}$

86. **Sine Rule :** $\frac{a}{SinP} = \frac{b}{SinQ} = \frac{c}{SinR}$

87. $\sqrt{\text{Area of square}} = \frac{\text{Perimeter of square}}{4} = \frac{\text{Diagonal of square}}{4} = \text{side of square}$



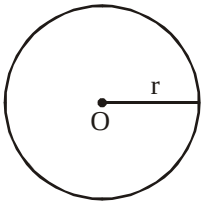
Square

88. In a circle with radius r.

$\frac{A}{C} = \frac{D}{4}$ where A - Area of circle

C - Circumference of circle

D - Diameter of circle

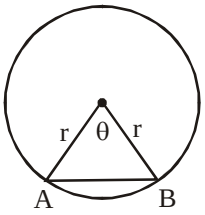


89. If $\theta = 60°$, ar $\Delta AOB = \frac{\sqrt{3}}{4}r^2$

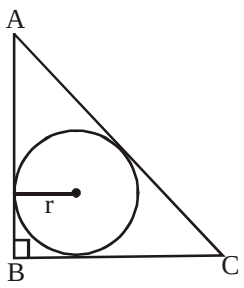
If $\theta = 90°$, ar $\Delta AOB = \frac{1}{2}r^2$

If θ, ar $\Delta AOB = \frac{1}{2}r^2$

$\sin\theta = r^2 \sin\left(\frac{\theta}{2}\right).\cos\left(\frac{\theta}{2}\right)$



90. (i) A circle with largest area inscribed in a right angle triangle, then $r = \frac{2 \times \text{area of } \Delta ABC}{\text{Perimeter of } \Delta ABC}$.



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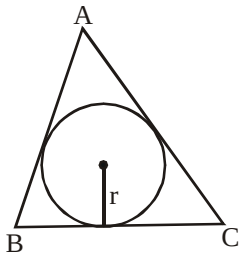
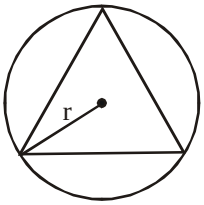
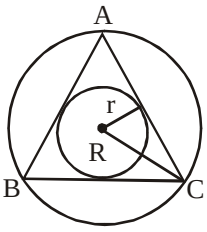
(ii) If ABC is an equilateral triangle with side a, then Area of circle = $\frac{\pi a^2}{12}$



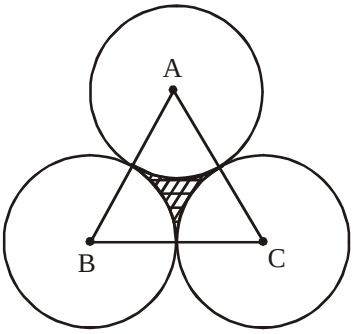
(iii) If ABC is an equilateral triangle with side a, then area of circle = $\frac{\pi a^2}{3}$.



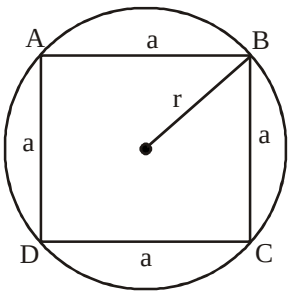
(iv) If ΔABC is an equilateral triangle, and two circles with radius r and R, then $\frac{r}{R} = \frac{1}{2}$ and $\frac{\pi r^2}{\pi R^2} = \frac{1}{4}$



(v) Three equal circle with radius r and an equilateral triangle ABC, then area of shaded region = $\left(2\sqrt{3} - \pi\right).\frac{r^2}{2}$



91. ABCD is a square placed inside a circle with side a and radius of circle r, then $\frac{\text{area of square}}{\text{area of circle}} = \frac{7}{11}$



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92. Diagonal of a cube = $\sqrt{3}$ × side

93. Diagonal of a cuboid = $\sqrt{l^2+b^2+h^2}$; where $\ell \rightarrow$ Length, b $\rightarrow$ breadth, h $\rightarrow$ height

94. For two cubes

$$\sqrt{\frac{A_1}{A_2}} = \sqrt[3]{\frac{v_1}{v_2}} = \frac{a_1}{a_2} = \frac{d_1}{d_2}$$

where A_1, $A_2 \rightarrow$ Area of cubes
v_1, $v_2 \rightarrow$ Volume
a_1, $a_2 \rightarrow$ Sides
d_1, $d_2 \rightarrow$ Diagonals

95. **Units of Measurement of Area and Volume**

- The inter-relationships between various units of measurement of length, area and volume are listed below for ready reference:

Length

1 Centimetre (cm)	=	10 milimetre (mm)
1 Decimetre (dm)	=	10 centimetre
1 Metre (m)	=	10 dm = 100 cm = 1000 mm
1 Decametre (dam)	=	10 m = 1000 cm
1 Hectometre (hm)	=	10 dam = 100 m
1 Kilometre (km)	=	1000 m = 100 dam = 10 hm
1 Myriametre	=	10 kilometre

Area

$1\ cm^2 = 1\ cm \times 1\ cm$	=	$10\ mm \times 10\ mm = 100\ mm^2$
$1\ dm^2 = 1\ dm \times 1\ dm$	=	$10\ cm \times 10\ cm = 100\ cm^2$
$1\ m^2 = 1\ m \times 1\ m$	=	$10\ dm \times 10\ dm = 100\ dm^2$
$1\ dam^2$ or 1 are	=	$1\ dam \times 1dam = 10\ m \times 10\ m = 100\ m^2$
$1\ hm^2$ = 1 hectare	=	$1\ hm \times 1\ hm = 100\ m \times 10000 m^2 = 100\ dm^2$
$1\ km^2 = 1\ km \times 1\ km$	=	$10\ hm \times 10\ hm = 100\ hm^2$ or 100 hectare

Volume

$1\ cm^3$ = 1 ml	=	$1\ cm \times 1\ cm \times 1\ cm = 10\ mm \times 10\ mm \times 10\ mm = 1000\ mm^3$
1 litre = 1000 ml	=	$1000\ cm^3$
$1\ m^3 = 1\ m \times 1\ m \times 1m$	=	$100\ cm \times 100\ cm \times 100\ cm = 10^6\ cm^3$
	=	1000 litre = 1 kilometre
$1\ dm^3 = 1000\ cm^3, 1m^3$	=	$1000\ dm^3, 1\ km^3 = 10^9\ m^3$

If a, b, c are the edges of a cuboid, then

96. The longest diagonal $= \sqrt{a^2+b^2+c^2}$
   (i) If the height of a cuboid is zero it becomes a rectangle.
   (ii) If "a" be the edge of a cube, then
   (iii) The longest diagonal = $a\sqrt{3}$

97. Volume of pyramid $= \frac{1}{3} \times$ Base Area $\times$ height (H)

98. (i) If A_1 & A_2 denote the areas of two similar figures and l_1 & l_2 denote their corresponding linear measures, then $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$

   (ii) If V_1 & V_2 denote the volumes of two similar solids and l_1, l_2 denote their corresponding linear measures, then $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$

   (iii) The rise or fall of liquid level in a container $= \frac{\text{Total volume of objects submerged or taken out}}{\text{Cross sectional area of container}}$

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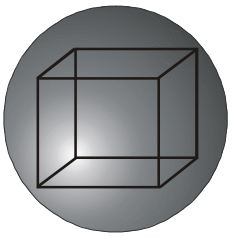
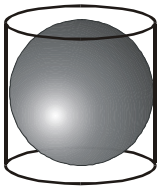
99. If a largest possible cube is inscribed in a sphere of radius 'a' cm, then

(i) the edge of the cube $= \frac{2a}{\sqrt{3}}$.



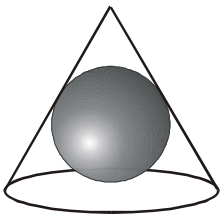
(ii) If a largest possible sphere is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then for h > a,

- the radius of the sphere = a and
- the radius $= \frac{h}{2}$ (for a > h)

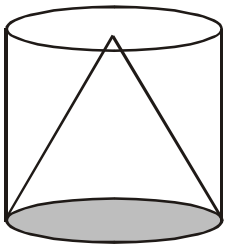


(iii) If a largest possible sphere is inscribed in a cone of radius 'a' cm and slant height equal to the diameter of the base, then

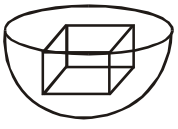
- the radius of the sphere $= \frac{a}{\sqrt{3}}$.



(iv) If a largest possible cone is inscribed in a cylinder of radius 'a' cm and height 'h' cm, then the radius of the cone = a and height = h.



(v) If a largest possible cube is inscribed in a hemisphere of radius 'a' cm, then the edge of the cube $= a\sqrt{\frac{2}{3}}$.



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100. In any quadrilateral

(i) $\text{Area} = \frac{1}{2} \times$ one diagonal × (sum of perpendiculars to it from opposite vertices) = $\frac{1}{2} \times d\,(d_1 + d_2)$

(ii) Area of a cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

where a, b, c, d are sides of quadrilateral and

s = semi perimeter $= \frac{a+b+c+d}{2}$

101. If length, breadth & height of a three dimensional figure increase/decrease by x%, y% and z%, then

$$\text{Change in area} = \left[\left(\frac{100 \pm x}{100}\right)\left(\frac{100 \pm y}{100}\right) - 1\right] \times 100\%$$

$$\text{Change in Volume} = \left[\left(\frac{100 \pm x}{100}\right)\left(\frac{100 \pm y}{100}\right)\left(\frac{100 \pm z}{100}\right) - 1\right] \times 100\%$$

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